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Tugas 4 PDP (F)

2.7 Selesaikan problem Cauchy $u_x + u_y = u^2$, $u(x,0) = 1$

Uji Penyelesaian Tunggal.

$$J = \begin{vmatrix} 1 & 1 \\ 0 & 1 \end{vmatrix} = 1 \neq 0 \quad \text{(maka, penyelesaian tunggal)}$$

Metode Karakteristik

- $\dfrac{dx}{dt} = 1$

$$dx = dt$$
$$\int dx = \int dt$$
$$x = t + f_1(s)$$

- $\dfrac{dy}{dt} = 1$

$$dy = dt$$
$$\int dy = dt$$
$$y = t + f_2(s)$$

- $\dfrac{du}{dt} = u^2$

$$\int \frac{du}{u^2} = \int dt$$
$$-u^{-1} = t + f_3(s)$$
$$-\frac{1}{u} = t + f_3(s)$$
$$u = -\frac{1}{t + f_3(s)}$$

Substitusi nilai awal untuk tentukan konstanta.

- $x(0,s) = 0 + f_1(s)$

$$S = f_1(s)$$
$$x(t,s) = t + S$$

- $y(0,s) = 0 + f_2(s)$

$$0 = f_2(s)$$
$$y(t,s) = t + 0 = t$$

- $u(0,s) = \dfrac{-1}{0 + f_3(s)}$

$$1 = -\frac{1}{f_3(s)}$$
$$f_3(s) = -1$$
$$u(t,s) = -\frac{1}{t-1}$$

Diperoleh :

$$y = t$$

Maka, penyelesaiannya, $u(x,y) = -\dfrac{1}{y-1}$

2.18 Perhatikan persamaan $uu_x + u_y = -\frac{1}{2}u$

a. Temukan solusi tunggal untuk kondisi $u(x, 2x) = x^2$

▶ Metode Karakteristik

$$\frac{dx}{dt} = u \; ; \; \frac{dy}{dt} = 1 \; , \; \frac{du}{dt} = -\frac{1}{2}u$$

$$x(0,s) = s \; ; \; y(0,s) = 2s \; ; \; u(0,s) = s^2$$

→ $\frac{dx}{dt} = u$

$dx = u\,dt$

$\int dx = \int u\,dt$

$x = ut + f_1(s).$

→ $\frac{dy}{dt} = 1$

$dy = dt$

$\int dy = \int dt$

$y = t + f_2(s)$

→ $\frac{du}{dt} = -\frac{1}{2}u$

$\frac{du}{u} = -\frac{1}{2}dt$

$\int \frac{du}{u} = \int -\frac{1}{2}dt$

$\ln u = -\frac{1}{2}t + f_3(s)$

$u = e^{-\frac{1}{2}t + f_3(s)}$

$= f_3(s).\, e^{-\frac{1}{2}t}$

▶ Substitusi nilai / kondisi awal

$x(0,s) = 0 + f_1(s)$

$s = f_1(s)$

$u(0,s) = f_3(s).(1)$

$s^2 = f_3(s)$

$y(0,s) = 0 + f_2(s)$

$2s = f_2(s)$

$x(t,s) = t + s$

$y(t,s) = t + 2s$

$u(t,s) = s^2 e^{-1/2\, t}$

$$\begin{aligned} x &= t + s \\ y &= t + 2s \\ \hline x - y &= -s \\ s &= y - x \end{aligned} \quad -$$

$$\begin{aligned} y &= t + 2s \\ 2x &= 2t + 2s \\ \hline y - 2x &= -t \\ t &= 2x - y \end{aligned} \quad -$$

$$u(x,y) = (y-x)^2 . e^{-\frac{1}{2}(2x-y)}$$

$$= (y-x)^2 e^{-x + \frac{1}{2}y}$$

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